

Umm Al-Qura University  
Faculty of Applied Science  
Department of Mathematical Science  
Saturday: 8 / 4 / 1440 H



Final exam  
First semester 1439/1440 H  
Math 4041101-4  
Time Limit: 2 hours

Differentiation and integration (1)

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**Student information:**

Name (Print): \_\_\_\_\_

Group (Print): \_\_\_\_\_

University ID (Print): \_\_\_\_\_

Serial No. (Print): \_\_\_\_\_

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This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and write your name on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any **calculator** on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **If you use a “fundamental theorem” you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Do **NOT** write in the table to the right.

1. (10 points) **Find**  $\frac{dy}{dx}$  for each of the following:

(a) (2 points)  $y = 2x^3 + 3x^2 + 5x$

(b) (2 points)  $y = (x^2 + 2)(x^3 + 1)$

(c) (2 points)  $y = \frac{3x-5}{x^2+7}$

(d) (2 points)  $y = (x^2 - x + 1)^5$

(e) (2 points)  $y = \sin(3x^2)$

2. (10 points)

(a) (6 points) Let  $f(x) = x^3 + 3x$ ,  $g(x) = \sqrt{x+3}$  and  $h(x) = \frac{1}{4x-1}$ .

(i) Find the following:

- the **natural domain** of  $f$
  
  
  
  
  
  
  
  
  
  
- the **natural domain** of  $g$
  
  
  
  
  
  
  
  
  
  
- the **natural domain** of  $h$
  
  
  
  
  
  
  
  
  
  
- $(f+g)(1)$
  
  
  
  
  
  
  
  
  
  
- $(f \circ g)(1)$

(ii) Show that the function  $f$  is **odd**.

(b) (4 points) Find each of the following limits

(i)  $\lim_{x \rightarrow 1^-} \frac{2x}{x-1}$

(ii)  $\lim_{x \rightarrow 5} \frac{x^2 - 25}{x - 5}$

(iii)  $\lim_{x \rightarrow -\infty} \frac{x^3}{2x^3 - 5x}$

(iv)  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^2 + 4}$

3. (10 points) Find each of the following:

(a) (2 points)  $\int_0^4 (3t^2 + \sqrt{t}) dt$

(b) (2 points)  $\int_{-1}^1 (x^3 + 3x) dx$

(c) (2 points)  $\int_0^{\pi/4} (\cos x - \sin x) dx$

(d) (2 points)  $\int (x^{\frac{1}{3}} + x^{\frac{3}{4}}) dx$

(e) (2 points)  $\int (x^3 + 6x)^5 (3x^2 + 6) dx$

4. (10 points) For the curve  $y = x^3 - 12x + 1$ , find the following:

(a) (2 points) the **critical** points,

(b) (2 points) the **increasing** and **decreasing** intervals,

(c) (2 points) the local **Maxima** and **Minima**,

(d) (2 points) the **concavity** intervals

(e) (2 points) the **inflection** point,

5. (10 points)

(a) (5 points) The function  $g$  is defined by

$$g(x) = \begin{cases} 7 - 3x & \text{if } x \leq 3, \\ 1 - \sqrt{3x} & \text{if } x > 3. \end{cases}$$

(i) (2 points) Find  $\lim_{x \rightarrow 3^-} g(x) =$

(ii) (2 points) Find  $\lim_{x \rightarrow 3^+} g(x) =$

(iii) (1 point) Is the function  $g$  **continuous** or **discontinuous**, at  $x = 3$  ?

(b) (2 points) Given that  $f(x) = \sin^2(x)$ , find

(i) (1 point)  $f'(x)$

(ii) (1 point)  $f''(0)$

(c) (3 points) For the curve  $x^2 + y^2 = 25$ , find the following:

(i) (2 points)  $\frac{dy}{dx}$

(ii) (1 point)  $\frac{d^2y}{dx^2}$

**Best regards.**